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B.M.S COLLEGE FOR WOMEN
BENGALURU – 560004

V SEMESTER END EXAMINATION – JAN/FEB-2024

B.Sc – MATHEMATICS
VECTOR DIFFERENTIAL CALCULUS AND ANALYTICAL GEOMETRY
(NEP Scheme 2021-22 onwards)

Course Code: MAT5DSC06 **QP Code:5025**
Duration: 2 ½ Hours **Max marks: 60**

Instructions: Answer all the sections.

SECTION-A

I. Answer any SIX of the following. Each question carries TWO marks. (6x2=12)

1. Define osculating plane and write its equation.
2. Find the cartesian polar coordinates of the point whose cylindrical coordinates are $\left(\frac{13}{5}, \frac{2\pi}{3}, \frac{3}{2}\right)$.
3. If $\emptyset = x^2 - y^2 + 4z$, find $\nabla^2\emptyset$.
4. Show that $\vec{F} = 2x^2z\hat{i} - 10xyz\hat{j} + 3xz^2\hat{k}$ is solenoidal.
5. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem.
6. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz\, dxdydz$.
7. Find the length of the perpendicular from (1,3,4) to the plane $\boldsymbol{2x - y + z + 3 = 0}$.
8. Obtain the equation of the sphere described on the join of **A(2,-3,4)** and **B(-5,6,-7).**

SECTION-B

II. Answer any FOUR of the following. Each question carries SIX marks. (4x6=24)

1. State and prove Serret -Frenet formulae.
2. Find the Curvature (ℵ) $and\ torsion(\tau)$ for the space curve $x = t - \frac{t^3}{3}$, $y = t^2$, $z = t + \frac{t^3}{3}$.
   surface $x^2 + y^2 + z^2 - 25 = 0$.
3. Express the vector $\vec{f} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ in terms of spherical polar coordinates , and find $f_r f_\theta, f_\emptyset$.
4. If $\vec{f}\ and\ \vec{g}$ are two vector fields, then Prove that $div(\vec{f} \times \vec{g}) = \vec{g} \cdot curl\vec{f} - \vec{f} \cdot curl\vec{g}$.
5. Find the directional derivative of $\emptyset(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1,1, -1) in the direction of $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

6. If $\vec{f} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$, $\vec{g} = 2y\hat{i} + 3z\hat{j} + 4x\hat{k}$ and $\vec{g} = 2z\hat{i} + 3x\hat{j} + 4y\hat{k}$
i) Find $\nabla(\vec{f} \cdot \vec{g})$ ii) Find $\nabla[\vec{f}\ \vec{g}\ ]$.

SECTION-C

III. Answer any FOUR of the following. Each question carries SIX marks. (4x6=24)

1. Verify Green's theorem in the plane for $\oint(xy + y^2)dx + x^2dy$ where c is the curve bounded by $y = x$ and $y = x^2$.
2. Verify Gauss Divergence theorem for $F = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelepiped bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 2$ and $z = 3$.
3. Evaluate by Stoke's theorem $\oint sinz\ dy - cosx\ dy + siny\ dz$, where C is the boundary of rectangle $0 \le x < \pi$, $0 \le y \le 1$, $z = 3$.
4. Derive the equation to find the length of the perpendicular from a point to the plane.
5. Find the length and the equations of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.
6. Show that the spheres $x^2 + y^2 + z^2 - 7x - 3y + 5x - 6 = 0$ and $2(x^2 + y^2 + z^2) + x + 8y + 9z + 19 = 0$ cut orthogonally. Find their plane of intersections.

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